

# Vacuum Condensates in the Global Color Symmetry Model

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## Abstract

Based on the quark propagator in the instanton dilute liquid approximation, we calculate analytically the quark condensate  $\langle \bar{q}q \rangle$ , the mixed quark gluon condensate  $g_s \langle \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q \rangle$  and the four quark condensate  $\langle \bar{q}\Gamma q\bar{q}\Gamma q \rangle$  at the mean field level in the framework of global color symmetry model. The numerical calculation shows that the values of these condensates are compatible with the ranges determined by other nonperturbative approaches. Moreover, we find that for nonlocal four quark condensate the previous vacuum saturation assumption is not a good approximation even at the mean field level.

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The non-perturbative structure of QCD vacuum is characterized by the vacuum matrix elements of various singlet combination of quark and gluon fields, especially by such condensates as the quark condensate  $\langle \bar{q}q \rangle$ , the mixed quark gluon condensate  $g_s \langle \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q \rangle$  and the four quark condensate  $\langle \bar{q}\Gamma q\bar{q}\Gamma q \rangle$ . These condensates would vanish in a perturbative vacuum but would not in the non-perturbative QCD vacuum, and are of essence for describing the physics of strong interaction at low and intermediate energies. Naturally, it is interesting to determine the vacuum values at least of the lower dimensional quark and gluonic operators.

Previous studies of the condensates of  $\langle \bar{q}q \rangle$ ,  $g_s \langle \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q \rangle$  and  $\langle \bar{q}\Gamma q\bar{q}\Gamma q \rangle$  include QCD sum rules where the condensates were treated as fit parameters in the analysis of various hadron channels [1-3], quenched lattice QCD[4], the instanton liquid model[5] and the effective quark-quark interaction model[6]. In the present letter we shall investigate the vacuum condensates in the framework of the global color symmetry model(GCM)[7-9] which is based on effective quark-quark interaction and can be defined through a truncation of QCD as follows. The generating functional for QCD in the Euclidean metric is

$$Z[\bar{\eta}, \eta] = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}\bar{\omega}\mathcal{D}\omega\mathcal{D}A \exp \left\{ -S - S_{gf} - S_g + \int d^4x (\bar{\eta}q + \bar{q}\eta) \right\}, \quad (1)$$

where

$$S = \int d^4x \left\{ \bar{q} \left[ \gamma_\mu \left( \partial_\mu - ig \frac{\lambda^a}{2} A_\mu^a \right) \right] q + \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \right\},$$

and  $S_{gf}$ ,  $S_g$  are the gauge-fixing and ghost actions. As a bilocal field  $B^\theta(x, y)$  being introduced, the generating functional can be given as

$$\begin{aligned} Z[\bar{\eta}, \eta] &= \exp \left[ W_1 \left( ig \frac{\delta}{\delta\eta(x)} \frac{\lambda^a}{2} \gamma_\mu \frac{\delta}{\delta\bar{\eta}(x)} \right) \right] \\ &\times \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}B^\theta(x, y) \exp \left\{ -S[\bar{q}, q, B^\theta(x, y)] + \int d^4x (\bar{\eta}q + \bar{q}\eta) \right\}, \end{aligned} \quad (2)$$

where

$$W_1[J_\mu^a] = \sum_{n=3}^{\infty} \frac{1}{n!} \int dx_1 \cdots dx_n D_{\mu_1 \cdots \mu_n}^{a_1 \cdots a_n}(x_1, \cdots, x_n) \Pi_{i=1}^n J_{\mu_i}^{a_i}(x_i),$$

and

$$S[\bar{q}, q, B^\theta(x, y)] = \int \int d^4x d^4y \left[ \bar{q}(x) G^{-1}(x, y; [B^\theta]) q(y) + \frac{B^\theta(x, y) B^\theta(y, x)}{2g^2 D(x - y)} \right],$$

with

$$G^{-1}(x, y; [B^\theta]) = \gamma \cdot \partial \delta(x - y) + \frac{1}{2} \Lambda^\theta B^\theta(x, y). \quad (3)$$

Here  $\Lambda^\theta = K^a C^b F^c$  is determined by Fierz transformation in the color, flavor and Lorentz space [7], and  $g^2 D(x-y)$  is the effective gluon propagator in GCM.

Neglecting  $W_1[J_\mu^a]$  we obtain the GCM generating functional

$$Z_{GCM}[\bar{\eta}, \eta] = \int \mathcal{D}\bar{q} \mathcal{D}q \mathcal{D}B^\theta(x, y) \exp \left\{ -S[\bar{q}, q, B^\theta(x, y)] + \int d^4x (\bar{\eta}q + \bar{q}\eta) \right\}. \quad (4)$$

Performing the functional integral over  $\mathcal{D}\bar{q}$  and  $\mathcal{D}q$  in Eq. (4), we have

$$Z_{GCM}[\bar{\eta}, \eta] = \int \mathcal{D}B^\theta(x, y) \exp(-S[\bar{\eta}, \eta, B^\theta(x, y)]), \quad (5)$$

where

$$\begin{aligned} S[\bar{\eta}, \eta, B^\theta(x, y)] &= -\text{Tr} \ln \left[ \not{\partial} \delta(x-y) + \frac{1}{2} \Lambda^\theta B^\theta(x, y) \right] \\ &+ \int \int \left[ \frac{B^\theta(x, y) B^\theta(y, x)}{2g^2 D(x-y)} + \bar{\eta}(x) G(x, y; [B^\theta]) \eta(y) \right]. \end{aligned} \quad (6)$$

The saddle-point of the action is defined as  $\delta S[\bar{\eta}, \eta, B^\theta(x, y)] / \delta B^\theta(x, y) \big|_{\eta=\bar{\eta}=0} = 0$  and is given by

$$B_0^\theta(x-y) = g^2 D(x-y) \text{tr}_{\gamma C} [\Lambda^\theta G_0(x-y)], \quad (7)$$

where  $G_0$  stands for  $G[B_0^\theta]$ . These configurations are related to vacuum condensates and provide self-energy dressing of the quarks through the definition  $\Sigma(p) \equiv \frac{1}{2} \Lambda^\theta B_0^\theta(p) = i\gamma \cdot p [A(p^2) - 1] + B(p^2)$ . The self energy functionals  $A(p^2)$  and  $B(p^2)$  are determined by the rainbow Dyson-schwinger equations[10]

$$[A(p^2) - 1]p^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{A(q^2) p \cdot q}{q^2 A^2(q^2) + B^2(q^2)}, \quad (8)$$

$$B(p^2) = \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)}. \quad (9)$$

This dressing comprises the notion of constituent quarks by providing a mass  $M(p^2) = B(p^2)/A(p^2)$ , reflecting a vacuum configuration with dynamically broken chiral symmetry. Because the form of the gluon propagator  $g^2 D(s)$  in the IR region remains missing. One often uses model forms as input in the previous studies of the rainbow Dyson-Schwinger equations [6-10]. On the other hand, much evidence of strong interaction phenomena is recently in favour of the instanton structure of the vacuum in QCD [11-14]. In particular, the complex configuration of the vacuum

of QCD with different topological winding numbers can be constructed in the model of an instanton dilute liquid [11,12]. Lattice QCD [15] calculations are also consistent with the above point of view. Therefore, contrary to the previous philosophy, we use the quark propagator in the model of the instanton dilute liquid instead of the gluon propagator as input (more details can be seen in Ref.[16]). In Ref.[16], we showed that the self energy function  $A(q^2) = 1$  and  $B(q^2)$  is equivalent to the dynamical mass  $m(q^2)$  of quarks in the instanton dilute liquid approximation, i.e.

$$B(q^2) = m(q^2) = \frac{\epsilon \bar{\rho}}{6} q^2 \varphi^2(q) , \quad (10)$$

where  $\varphi(q) = \pi \bar{R}^2 \frac{d}{dz} [I_0(z)K_0(z) - I_1(z)K_1(z)]$  with  $z = \frac{|q|\bar{R}}{2}$ .  $I_n(z)$  ( $K_n(z)$ ) ( $n = 0, 1$ ) are the first (second) kind modified Bessel functions of order  $n$ .  $\bar{\rho} \approx (200 \text{ MeV})^4$  is the average density of the instantons,  $\bar{R} = \frac{1}{3} \text{ fm}$  the average radius of the instantons, and  $\epsilon$  a constant  $(85 \text{ MeV})^{-1}$ .

Since the self energy function  $A(q^2)$  and  $B(q^2)$  have been determined in the instanton dilute liquid approximation we can calculate the vacuum condensates by the above saddle-point expansion, that is, we shall work at the mean field level.

According to the definition of the GCM generating functional it is now straightforward to calculate the vacuum expectation value of any quark operator of the form

$$O_n \equiv (\bar{q}_{j1} \Lambda_{j1i1}^{(1)} q_{i1}) (\bar{q}_{j2} \Lambda_{j2i2}^{(2)} q_{i2}) \cdots (\bar{q}_{jn} \Lambda_{jnin}^{(n)} q_{in}) \quad (11)$$

in the mean field vacuum. Here the  $\Lambda^{(i)}$  stands for an operator in the Dirac, flavor and color space.

Taking the appropriate value of derivatives of Eq.(5) with respect to external sources  $\eta_i$  and  $\bar{\eta}_j$  (putting  $\eta_i = \bar{\eta}_j = 0$  [17]), we have

$$\langle : O_n : \rangle = (-)^n \sum_p [(-)^p \Lambda_{j1i1}^{(1)} \cdots \Lambda_{jnin}^{(n)} (G_0)_{i1jp(1)} \cdots (G_0)_{injp(n)}] \quad (12)$$

where  $p$  stands for a permutation of the  $n$  indices. In particular, we obtain the nonlocal quark condensate  $\langle : \bar{q}(x)q(0) : \rangle$

$$\langle : \bar{q}(x)q(0) : \rangle_\mu = (-) \text{tr}_{\gamma C} [G_0(x, 0)]$$

$$\begin{aligned}
&= (-4N_c) \int_0^\mu \frac{d^4p}{(2\pi)^4} \frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)} e^{ipx} \\
&= (-) \frac{12}{16\pi^2} \int_0^\mu ds s \frac{B(s)}{s A^2(s) + B^2(s)} \left[ 2 \frac{J_1(\sqrt{s x^2})}{\sqrt{s x^2}} \right], \tag{13}
\end{aligned}$$

where  $\mu$  is the renormalization scale which we choose to be  $1 \text{ GeV}^2$ . At  $x=0$  the expression for the local condensate  $\langle: \bar{q}q : \rangle$  is reproduced

$$\langle: \bar{q}q : \rangle_\mu = (-) \text{tr}_{\gamma_C} [G_0(x, 0)]|_{x=0} = (-) \frac{12}{16\pi^2} \int_0^\mu ds s \frac{B(s)}{s A^2(s) + B^2(s)}. \tag{14}$$

The nonlocality  $g(x^2)$  can be obtained immediately by dividing Eqs.(13) by Eq.(14).

Table. I. The nonlocal quark condensate  $g(x) = \langle: \bar{q}(x)q(0) : \rangle / \langle: \bar{q}(0)q(0) : \rangle$ .

$x^2(\text{GeV}^2)$	0.0	2.0	4.0	6.0	8.0	10	12	14	16	18	20
$g(x^2)$	1.0	0.89	0.80	0.71	0.64	0.57	0.50	0.45	0.40	0.35	0.31

In Table I we display the nonlocal quark condensate  $g(x)$  versus  $x^2$ . By comparing it with the corresponding results mentioned in the Ref.[18] we demonstrate that the nonlocal quark condensate is very robust in our approach or the method of model gluon propagators.

Another important vacuum condensate following from Eq.(11) is the nonlocal four quark condensate in the mean field vacuum. In the case of  $\Lambda^{(1)} = \Lambda^{(2)} = \gamma_\mu \frac{\lambda_C^a}{2}$  we find

$$\begin{aligned}
&\langle: \bar{q}(x) \gamma_\mu \frac{\lambda_C^a}{2} q(x) \bar{q}(0) \gamma_\mu \frac{\lambda_C^a}{2} q(0) : \rangle_\mu \\
&= -\text{tr}_{\gamma_C} [G_0(0, x) \gamma_\mu \frac{\lambda_C^a}{2} G_0(x, 0) \gamma_\mu \frac{\lambda_C^a}{2}] + \text{tr}_{\gamma_C} [G_0(x, x) \gamma_\mu \frac{\lambda_C^a}{2}] \text{tr}_{\gamma_C} [G_0(0, 0) \gamma_\mu \frac{\lambda_C^a}{2}] \\
&= (-) \int_0^\mu \int_0^\mu \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} e^{ix \cdot (p-q)} \left[ 4^3 \frac{B(p^2)}{A^2(p^2)p^2 + B^2(p^2)} \frac{B(q^2)}{A^2(q^2)q^2 + B^2(q^2)} \right. \\
&\quad \left. + 2 \times 4^2 \frac{A(p^2)}{A^2(p^2)p^2 + B^2(p^2)} \frac{A(q^2)}{A^2(q^2)q^2 + B^2(q^2)} p \cdot q \right]. \tag{15}
\end{aligned}$$

Similarly, at  $x=0$  the expression for the local four quark condensate  $\langle: \bar{q} \gamma_\mu \frac{\lambda_C^a}{2} q \bar{q} \gamma_\mu \frac{\lambda_C^a}{2} q : \rangle$  is recovered:

$$\langle: \bar{q} \gamma_\mu \frac{\lambda_C^a}{2} q \bar{q} \gamma_\mu \frac{\lambda_C^a}{2} q : \rangle_\mu = (-4^3) \left[ \int_0^\mu \frac{d^4p}{(2\pi)^4} \frac{B(p^2)}{A^2(p^2)p^2 + B^2(p^2)} \right]^2 = (-) \frac{4}{9} \langle: \bar{q}q : \rangle^2, \tag{16}$$

namely, for the local four quark condensate, our result is consistent with the vacuum saturation assumption of Ref.[1]. However, if we consider the nonlocal four quark condensate, it should be noted that the contribution of the second term of right-hand of Eq.(15) can not be neglected.

As to the mixed condensate  $g_s < \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q >$ , we can use the method described by Ref.[6] to obtain the mixed condensate in Minkowski space

$$g_s < \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q >_{\mu} = (-)(\frac{N_c}{16\pi^2})\{\frac{27}{4}\int_0^\mu ds s \frac{B[2A(A-1)s+B^2]}{A^2s+B^2} + 12\int_0^\mu ds s^2 \frac{B(2-A)}{A^2s+B^2}\}. \quad (17)$$

In Table II we display the result for  $< \bar{q}q >$  and  $g_s < \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q >$  in our approach and compare it with the corresponding values which were obtained from other nonperturbative approaches (QCD sum rules[3], quenched lattice QCD[4], the instanton liquid model[5] and the effective quark-quark interaction model[6]).

Table. II,  $< \bar{q}q >$  and  $g_s < \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q >$  in different non-perturbative approaches

	$(-)< \bar{q}q >^{\frac{1}{3}}$	$(-)< g_s \bar{q}\sigma G q >^{\frac{1}{5}}$
present work	207 MeV	415 MeV
QCD sum rules[3]	210 –230 MeV	375 –395 MeV
quenched lattice[4]	225 MeV	402 –429 MeV
instantion liquid model[5]	272 MeV	490 MeV
effective quark-quark interaction model[6]	150 –180 MeV	400 –460 MeV

Table II shows that our results for  $< \bar{q}q >$  and  $g_s < \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q >$  are compatible with the ranges obtained from other nonperturbative methods, especially from QCD sum rules[3] and quenched lattice[4]. It should be noted that there is not any free parameter in our model. In this model, the calculated masses, decay constants of the mesons  $\pi$  and  $\sigma$  and decay width of  $\sigma \rightarrow \pi\pi$  agree with experimental data quite well[16].

In summary, based on the instanton dilute liquid approximation, we have determined the quark condensate  $< \bar{q}q >$ , the mixed quark gluon condensate  $g_s < \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q >$  and the four quark condensate  $< \bar{q}\Gamma q \bar{q}\Gamma q >$  at the mean field level in the framework of GCM. The numerical calculation have shown that our results are compatible with the ranges obtained from other nonperturbative approaches. Futhermore, we have found that the contribution of the second term of right-hand of Eq.(15) must be taken into account when one calculates nonlocal four quark condensate. This implies that even at the mean field level the previous vacuum saturation assumption is not a good approximation for the nonlocal four quark condensate.

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